MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2017/2018

TMA 1111 – MATHEMATICAL TECHNIQUES

(All sections / Groups)

12 OCTOBER 2017
2.30pm-4.30pm
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This Question paper consists of **6 pages** only excluding the cover page.

2. There are **2 Sections**:
   Section A (4 Questions): Attempt **ALL FOUR (4)** questions.
   Section B (2 Questions): Attempt **ONE (1)** question **ONLY**.
   All questions carry equal marks and the distribution of the marks for each question is given.

3. Please write your answers in the Answer Booklet provided, and **start each question on a new page.**

4. Show all steps.

5. Formulas are provided and attached in Appendix.

SECTION A

Answer ALL FOUR (4) Questions.

Question 1 (LO1)

a) Solve the system of linear equations by using the inverse of a matrix:

$$x + y + z = 6$$
$$2y + 5z = -4$$
$$2x + 5y - z = 27$$

[7 marks]

b) Two anchors are holding a ship in place and their forces acting on the ship are represented by vectors A and B as follow:

$$A = 12i + 6j - 2k \quad \text{and} \quad B = -8i + 5j + 2k$$

If we were to replace the 2 anchors with 1 single anchor, what vector represents that single vector?

[3 marks]

Question 2 (LO1)

a) Compute real and imaginary part of $z = \dfrac{2i-2}{2i-3}$ **[3 marks]**

b) By using De Moivre's Formula simplify the following problem and leave the answer in Euler's form:

$$\frac{(\cos 4\theta + i\sin 4\theta)^2(\cos 2\theta + i\sin 2\theta)^4}{(\cos 4\theta + i\sin 4\theta)^2(\cos(-\theta) + i\sin(-\theta))^4}$$

[4 marks]

c) Find the polar form of $z_3 = 2\sqrt{3} + 2i$ **[3 marks]**

Continued...

Question 3 (LO1)

a) Solve $\frac{dy}{dx} = \frac{3x^2}{4y}$ by using the separable method. **[2 marks]**

b) If p, q, r, s are constants, under what conditions is $(px+qy)dx+(rx+sy)dy=0$ exact? Then, solve the exact equation. **[4 marks]**

c) Find the general solution of the following non-homogeneous linear differential equation:

$$y''+y=\sin 2x\,.$$ **[4 marks]**

Question 4 (LO2)

A study was carried out to investigate whether there is a relationship between the number of online shopping transactions per day, x, and the number of customer reviews received in a day, y. The summary of the sample data collected is as follow:

$$n=10,\ \sum x=230,\ \sum y=31,\ \sum x^2=6618,\ \sum y^2=109,\ \sum xy=827$$

a) Compute $\bar{x}$, $\bar{y}$, S_{xy} and S_{xx}. **[4 marks]**

b) Find the regression model, $\hat{y}=\beta_0+\beta_1 x$, to fit the data. **[3 marks]**

c) Calculate the correlation coefficient, r. Then, infer the value of r. **[3 marks]**

Continued...

SECTION B

Answer ONE(1) Question ONLY.

Question 5 (LO2)

a) Find the value of f '(x) for each of these functions by using Chains' Rule:

i. $f(x) = \sqrt{\sin 5x}$

ii. $f(x) = \frac{4}{\ln 3x}$

iii. $f(x) = e^{\cos(6x-1)}$

[6 marks]

b) What is the largest possible area of a rectangular field that can be enclosed by 200m of fencing as in Figure 1 below. (Hint: Use the formula for a rectangular perimeter)

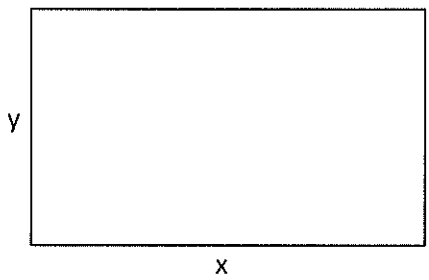

Figure 1: Rectangular field

[4 marks]

Continued...

Question 6 (LO2)

a) Evaluate $\int_0^4 (\sqrt{x}+1)-\left(\frac{2+x}{2}\right)dx$. **[2 marks]**

b) Use integration by substitution to solve $\int \frac{3e^{3x}}{\sqrt{1+2e^{3x}}}dx$. **[4 marks]**

c) Use integration by parts to show that $\int \ln x dx = x\ln x - x + C$. **[4 marks]**

Continued...

APPENDIX

1. Length of a vector v : $\|\mathbf{v}\| = \sqrt{v_1^2 + v_2^2}$

2. $\mathbf{u} \cdot \mathbf{v} = \|\mathbf{u}\| \|\mathbf{v}\| \cos\theta$

3. Dot product: $\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$

4. Parametric equation: $x = x_0 + at$; $y = y_0 + bt$; $z = z_0 + ct$

5. Criterion for an Exact Differential Equation : $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

6. General solution of the homogeneous linear second order differential equation

   For Distinct real roots: $y = c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x}$

   For Repeated roots: $y = c_1 e^{\lambda_1 x} + c_2 x e^{\lambda_2 x}$

   For Complex conjugate roots: $y = e^{\alpha x} (c_1 \cos \beta x + c_2 \sin \beta x)$.

7. Derivative Formulas

$\frac{d}{dx} e^x = e^x$
$\frac{d}{dx} \ln x = \frac{1}{x}$
Power Rule: $\frac{d}{dx} x^n = n x^{n-1}$ if $f(x) = x^n$ with $n \in R$
Product Rule: $\frac{d}{dx}(f \cdot g) = f \frac{dg}{dx} + g \frac{df}{dx}$
Chain Rule: If $y = f(u)$ and $u = g(x)$, then $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

Continued...

8. Integration Formulas

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$, for $n \neq -1$, n rational
$\int \frac{1}{x} dx = \ln x + C$
$\int e^x dx = e^x + C$
$\int \sin kx dx = -\frac{\cos kx}{k} + C$
$\int \cos kx dx = \frac{\sin kx}{k} + C$
Integration by Substitution: If $u = g(x)$ and $du = g'(x)dx$, then $\int f(u)du = \int f(g(x))\frac{d}{dx}g(x)dx$
Integration by Part: $\int udv = uv - \int vdu$

9. $\bar{x} = \frac{\sum x}{n}$

$$S_{xy} = \sum xy - \frac{\left(\sum x \sum y\right)}{n}, \qquad S_{xx} = \sum x^2 - \frac{\left(\sum x\right)^2}{n}, \qquad S_{yy} = \sum y^2 - \frac{\left(\sum y\right)^2}{n}$$

10. $\beta_1 = \frac{S_{XY}}{S_{XX}}$, $\qquad \beta_0 = \bar{y} - \beta_1 \bar{x}$

11. Sample correlation coefficient = $r = \frac{S_{xy}}{\sqrt{S_{xx}}\sqrt{S_{yy}}}$

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